Exercise II

1. Let R be a commutative unital ring. Prove that
   (a) every maximal ideal of R is prime,
   (b) the radical $\sqrt{I}$ of an ideal I of R is an ideal,
   (c) every prime ideal of R is radical,
   (d) an ideal $\mathfrak{m}$ of a ring R is maximal if and only if the ring $R/\mathfrak{m}$ is a field,
   (e) an ideal $\mathfrak{p}$ of a ring R is prime if and only if the ring $R/\mathfrak{p}$ is a domain,
   (f) an ideal $\mathfrak{I}$ of a ring R is radical if and only if the ring $R/\mathfrak{I}$ is reduced.
2. Let $F : V \longrightarrow W$ be a morphism of affine algebraic varieties. Prove that F is continuous in the Zariski topology.
3. Identify the prime ideals and reduced ideals in the rings:
   (a) $\mathbb{Z}$,
   (b) $\mathbb{Z}/(8\mathbb{Z})$,
   (c) $\mathbb{Z}/(4\mathbb{Z})$.
4. Which ideals in 3(a), 3(b) and 3(c) above are reduced but not prime?