Exercise, Set III

1. Let $X \subset k^n$. Show that any ideal of the form $I(X)$ is a radical ideal.

2. Describe reducibility of the variety $V(f) \subset k^n$ where k is algebraically closed and $f \in k[x_1, \cdots, x_n]$ a hypersurface.

3. Let $V(I) \subset \mathbb{A}^3$ be the algebraic set corresponding to the ideal

$$I := (x^2 - yz, xz - x) \subset k[x, y, z].$$

   Decompose $V(I)$ into its irreducible components.

4. Explain why Hilbert's Nullestellensatz fails for fields which are not algebraically closed.