Exercises, Set IV

1. Show that the pullback of $f : V \to W$ is a k-algebra homomorphism.
2. Determine the coordinate ring of the line $l := \{(x, y) \ : \ y - ax - b = 0\} \subset k^2$.
3. Find the coordinate ring of the cone $V(x^2 + y^2 - z^2)$ in $\mathbb{C}^3$.
4. Show that the two polynomials $f, g \in \mathbb{C}[x, y, z]$ defined by $f(x, y, z) = x^3 + 2xy^2 - 2xz^2 + x$ and $g(x, y, z) = x - x^3$ are the same considered as elements of $\mathbb{C}[V]$ where V is the variety in Question 3 above.